TECHNICAL NOTES.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

No. 16.

EXPERIENCE WITH GEARED PROPELLER DRIVES FOR AVIATION ENGINES.

By
K. Kutzbach.

Translated from
Technische Berichte Vol. III.- Sec. 3,

by

Starr Truscott, Aeronautic Engineer,
Bureau of Construction & Repair, U.S.N.

September, 1920.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

TECHNICAL NOTE NO. 16.

EXPERIENCE WITH GEARED PROPELLER DRIVES FOR AVIATION ENGINES.*

By

K. Kutzbach.

Translated from Technische Berichte, Vol. III, Sec. 3,
by Starr Truscott, Aeronautic Engineer,
Bureau of Construction & Repair, U. S. N.

SUMMARY.

I. THE DEVELOPMENT OF THE GEAR WHEELS.

(a) Bending stresses.

The greatest stress in the gear tooth is determined by the value of $\frac{P_{max}}{b.t}$. If one assumes - as is common with straight cut spur gears, - that the greatest tooth pressure (peripheral pressure) encountered, P_{max}, is distributed uniformly over the whole width b, but is carried only by the outer corner of one tooth, then for gear wheels with teeth of the common involute form

$$K_b \sim 14 \cdot \frac{P_{max}}{b.t} \qquad (1)$$

* (An expansion of a report sent in as an introduction to a discussion on experience with geared propeller drives held on May 10, 1918, at Charlottenburg.)

From the power N delivered by the engine to the gear there can be determined of course only the mean value $\frac{P_u}{b.t}$, in which $P_u = \frac{75.N}{V_u}$, and this value has been computed for the various captured engines which were studied, Table I. P_{max} can under certain circumstances be considerably greater than P_u, either because of acceleration pressures resulting from incorrect pitch or form of teeth, or because of irregular delivery of power from the engine, or finally because of reinforced vibration near a resonance period of the shaft; consequently, no statements can be made as to the actual magnitudes of P_{max}. Accordingly, in Table I, there are considered only the mean tooth pressures (peripheral pressures) P_u, computed from the engine powers.

From Table I it can be concluded that with good steel one can at once assume $\frac{P_u}{bt} = 200$, although this value according to formula (1) represents a stress $K_b = 2800$ kg/cm^2. With somewhat more accurate pitching the load is carried by more than the one tooth, because of the deformation of the loaded tooth. The theoretical stress in the teeth would be less still if the root and tip of the tooth were not made so high, as, for instance, in the Napier gear (in which to be sure the overlapping in meshing is reduced), or if the root were made specially thick (as, for instance, by the Maag, Friedrichshafen). This becomes especially true with the use of oblique teeth (as in the Hispano-Suiza), to which class belong herring bone gears and arc-shaped gears, since in these the tooth pressure is distributed uniformaly on an oblique line running from the root to the tip of the tooth. The stresses are worse, however, if the teeth bear unevenly as a result, for instance, of warping in hardening, untrue keying or poor forming. Too small a radius of the root is a more common defect, and on account of the scoring action is very dangerous. All these circumstances must be considered in determining the bending stress or the value of $\frac{P_u}{bt}$.

It was determined that chrome nickel steel was the material used in most of the gear wheels of the captured engines. The gears are hardened (case hardened) as a rule but are not all ground.

(b) Compressive stresses.

In general, tooth failures rarely appear in the captured engines and in those instances where they have been found they might have occurred in landing. However, sufficient bending strength can be easily obtained even with straight cut gears.

The compressive strength, which might be called the bearing strength, seems more important; that is, the surface pressure of the opposing curved tooth faces must never exceed the elastic limit if no permanent deformation and consequently no wear of the teeth is to occur. The compressive strength also has a direct effect on the preservation of the lubricating film between the teeth, for the greater the surface pressure and the smaller the relative velocity of sliding V_g of the teeth the more difficult to keep the oil between the teeth. The relative sliding speed of straight toothed gears is zero at the pitch or rolling circle, where pure rolling of the teeth on one another occurs. At this point the oil is easily squeezed out; metallic contact between the surfaces of the teeth occurs and if the elastic limit is exceeded, distortion or wear is unavoidable.

In order to compare the bearing strength of straight cut gears with the results of experience with bearings one can compute the relative tooth curvature of the teeth at the rolling circle. For involute teeth the radii of curvature of the teeth at the rolling circle are the distances e_1 and e_2 between the central point C and the tangent points G_1 and G_2 of the tangents to the base circles (see Fig. 1). The relative curvature of the teeth accordingly expresses the curvature of a roller lying on a plane and whose diameter δ_r is given by the equation:-

$$\frac{2}{\delta_r} = \frac{1}{e_1} \pm \frac{1}{e_2} = \frac{1}{r_1 \cdot \cos\alpha} \pm \frac{1}{r_2 \cos\alpha}$$

The + sign applies if the centers are on opposite sides of the axis (external or spur gears with involute system) the - sign if they are on the same side (internal gears). The values of $\frac{P_{max}}{b \cdot \delta_r}$ then are a measure of the "bearing strength." The values of the roller diameters δ_r for the captured engine gears are given in Table I and since P_{max} is unknown the values of $\frac{P_u}{b \cdot \delta_r}$ are given.

From experience with the gears it is concluded that if all the gears are of hardened steel

$$\frac{P_u}{b \cdot \delta_r} \leqq 100 \text{ should be suitable}$$

and that if $\frac{P_u}{b \cdot \delta_r} \leqq 30$ the gears need not be hardened. It is especially notable what favorable roller diameters are obtained with internal gears, with which hardening is as a rule unnecessary. For roller bearings where hardened rolls run between hardened rings

$\frac{P_u}{b \cdot \delta_r} = 200$ and more is permissible for low peripheral speeds. Where the rolls bear directly on the unhardened shaft 10 to 20 should be substituted.

With oblique toothed gears the contact shifts with great speed from side to side (with Herringbone and arced tooth gears from the center out and inversely, respectively) as a result of which the lubricating film is squeezed out with more difficulty. A small angle to the teeth is of advantage in this connection.

As to tooth forms, involute teeth are found in all the captured engines, which have the great advantage of being accurately formed and independent of center distances, although the Cycloidal type would have better fitting teeth and consequently smaller values of $\frac{P_u}{b \cdot \delta_r}$

(c) Heating.

When comparing gears besides the bending and crushing stresses their tendency to heat is important. The determining factors here are the heat generated - dependent on $\mu . P . V_g$ - and the surfaces of the gear wheels which absorb and carry away the heat. In this only the width and diameter of the gears are of importance quite independently of the pitch. V_g, the momentary sliding velocity of the teeth is given by

$$V_g = e(\omega_1 \pm \omega_2) = \frac{e(n_1 \pm n_2)}{9.6}$$

in which e is the distance from the central point C (Fig. 2) at which the teeth touch and the + sign applies for external (spur) gears and the - sign for internal. The distance e varies from 0 to maximum values which are dependent on the pitch t. The mean value of V_g is consequently dependent on t and the respective revolutions n_1 and n_2 of the gears. Accordingly, the expression

$$w = \frac{P_u . t (n_1 \pm n_2)}{b.d} = \frac{P_u}{b} . \frac{(n_1 \pm n_2)}{z}$$

can be taken as a measure of the heating of the gears.

(NOTE: If several gears (say i) work with one, as for instance in the Rolls Royce drive, where i equals successively 3, 1, and 3, then this equation becomes

$$w = i . \frac{P_u}{b} . \frac{(n_1 \pm n_2)}{z}.$$

The values are to be computed for all four gears in this manner.

In Table I this value is given for the smaller gears of each drive, since these give the higher values, and from experience with the gears it can be assumed that w $<$ 30,000 should be suitable. The larger w is the most convenient method of cooling the gears, - that is, the carrying off of the heat into metal parts and cooling these with air, - must be assisted by the less certain method of oil cooling. The smaller w the less need it be feared that the gears will run hot if the lubrication is temporarily interrupted. The Rolls-Royce gear (Nos. 11 to 13, Table 1) has the smallest value of w. This is lubricated, like the bearings, from the shaft and partially by oil from without, but the oil thrown off flows back into the "dry" crankcase. In the Wolseley Hispano-Suiza gear (No. 10) a very heavy lubrication is provided by a special oil pump which squirts the oil into the point where the teeth mesh through a slit in a pipe.

The gears should not be lubricated so heavily that the oil heats up as the result of a sort of churning. This may occur either because the walls of the case are too close to the gears or because the oil is caught between the faces of wide gears and forced out sidewise with great force. In either case unnecessary friction is produced with heating and thinning of the oil and a corresponding loss of power. The more the dissipation of the heat generated can be left to the metal parts and the air the better for the gear drive and its efficiency.

(d) Precision of Manufacture.

The peripheral speed V_u is a quantity frequently used in comparing gear wheels. It becomes more important the more defects there are in the transmission ratio due to inaccuracies in pitch or tooth forms. Inaccuracies in the teeth can be very plainly recognized with the Saurer gear testing machine. By courtesy of the Zahnradfabrik Friedrichshafen (Friedrichshafen gear factory) several diagrams from this machine are reproduced in Figs. 3 to 7.

(The Saurer gear testing machine tests gear wheels of any ratio. Two accurately circular pulleys, which are connected by a steel belt, provide an exact transmission without play or backlash with which the actual transmission of the gear wheels is compared. If the toothing is free from inaccuracies the pointer of the machine draws a circle or, if the pulleys have not the precise ratio of the gears, a spiral. The radial variations from the spiral correspond to tangential variations of the center distances magnified 200 times, and therefore show the angular error.)

The diagrams bring out in a striking manner the defects of the transmission which may be caused either by inaccurate setting of the gear wheels in manufacture or erection, or by inaccuracies in the dividing plate or gear cutting machine, but especially by shrinking when hardening. Besides these there occur in the gear drive itself errors due to bending of the shafts or shifting of the unequally heated gear wheels. Defects in the transmission ratio cause movements back and forth of the teeth, the blows from which are divided between both gears in proportion to the frictional and inertia resistance. Besides these there also occur reciprocal displacements of the gear centers as a result either of play in the bearings or springing of the shafts. An error of .01 mm. at the periphery would correspond to a center displacement four times as great or 0.04 mm. If the mass of one gear is very great compared to that of the other it will run on uniformly and the smaller will take all the variations. This applies for instance to propeller shafts whose revolving masses far exceed those of the crankshaft.

The magnitude of the acceleration pressures which arise in this case, can be comprehended if one considers the time in which the motion takes place. All the motions can be considered as portions of harmonic vibrations and thus made more convenient for computation as the computation of the acceleration pressures for these is very simple.

Assume, for instance, that a tooth movement is part of a harmonic vibration whose period is the 1/ith part of a revolution, The whole vibration then has a period of $T = \frac{60}{n_1 . i}$ sec. and the greatest acceleration pressure is $P = m_{r1} . a_1 . \omega_1^2$ in which m_{r1} is the mass rigidly attached to the teeth of gear I referred to the rolling circle, a_1 (in meters) is the radius of the vibration or the distance by which gear I departed from mean position, and

$$\omega_1 = \frac{2\pi}{T} = \frac{n_1 . i}{9.6}$$

is the angular velocity of the harmonic vibration. Expressing a_1 in mm.

$$\pm P = m_{r1} \frac{(\pm a_1)mm}{1000} = \left(\frac{n_1 . i}{9.6}\right)^2 \sim G_{r1} . a_1^{mm} . \left(\frac{n_1 i}{9.6}\right)^2$$

If now for example, $G_{r1} = 5$ kg., $a_1 = 0.05$ mm., n = 1800 r.p.m., z = 20, then $\pm P = \pm 400$ kg.

Consequently, coarse inaccuracies of tooth form or pitch can excite forces of considerable importance which are superposed on the forces transmitted from the engine. If in the preceding example the mean peripheral pressure is only occasionally $P_u < 400$ kg., changes in direction of pressure occur in the teeth and the gears will be dashed to and fro. For this reason small gears and high peripheral pressures are generally desirable. As for the rest it is easily seen what relation the acceleration pressures have to G_r.a, n and i. The practical method for reducing these added pressures is indicated, however, since n and i can generally be changed less easily.

1. Reduce the referred weight Gr of both gears or at least of one. The masses revolving with the gear teeth should be kept as small as possible, either by lightening the gears or by separating the teeth or the wheel from the other shaft by a flexible mounting. It will be explained later to what extent the flexible mounting may work unfavorably.

2. Reduce the inaccuracies a. As far as they arise from the pitching or the tools they are smaller, the smaller the diameter and the finer the pitch (or Modulus). But with fine pitches i is easily increased. On the other hand, the balancing effect of the lubricating oil becomes greater, the smaller a is, which is favorable to fine pitches (compare the Rolls-Royce gear). The best means, however, is to increase the requirements as to accuracy of toothing and assembly by accurate measurement of defects and a reduction of the magnitude of the allowable defects. If, for instance, in the example given above, the inaccuracy instead of being .05 mm. was only .005 mm. - which can easily be obtained by grinding - then $\pm P = \pm 40$ kg. which is quite permissible for the gears in question.

The diagrams of the Saurer gear testing machine show that the accuracy of the transmission ratio can be carried very far by grinding. Accordingly, the gears should either be milled or shaped with the greatest accuracy and used unhardened with correspondingly reduced stresses or hardened andground. In either case all gears should be tested in order that defects may be discovered in time.

The clearance (play) between the teeth may be as large as desired if, or as long as, no change in direction of pressure occurs. If, for instance, with a mean peripheral pressure of 400 kg. $P_{umax} = +$ 800 kg., and we never have $P_{umin} \gtreqless 0$, the teeth remain always in contact. However, changes in direction of pressure occur practically, with great irregularity of torque - for instance, with engines with a small number of cylinders or low revolutions, 1) - with very inaccurate toothing, but especially at periods of "critical vibration" which will be discussed later. It is satisfactory in any case if the teeth have a little play, which should be reduced only to avoid too much noise when idling and in the region of resonance. The gears of the Hispano-Suiza engines are assembled with a noticeable clearance so that after they have warmed up a sufficient oil clearance will remain.

1) Four 4-cycle cylinders (at 180°) cause reversals in direction of pressure before the fly wheel for all speeds of revolution, and consequently, in the gear wheels without fly wheels. Six cylinders have this effect only at a low torque or with very heavy moving masses. Consequently, a 6-cylinder airplane engine can cause reversals either at low r.p.m. near idling speeds or at very high r.p.m. (with heavy pistons and high piston speeds). With 5, 7, 8 and more cylinders with crank angles equally spaced, the piston masses have no effect on uniformity and reversals can occur near idling speeds only for a smaller number of cylinders. Naturally, the region of reversals is largely dependent upon the compression ratio, which affects the negative work.

The reason the mass effects play a part only in four and six cylinder engines lies in the peripheral forces excited in the crank circle by the mass effects.

$P_u = \sum (P_m \cdot \sin\alpha)$ in which P_m are the mass pressures of the reciprocating parts. Now $P_m = mr \cdot \omega^2 (\cos\alpha + \lambda \cos 2\alpha)$. Hence $P_u = \frac{r\omega^2}{2} \sum m \cdot (\sum \cos 2\alpha - \lambda \cdot \sum \sin\alpha + \lambda \cdot \sum \sin 3\alpha)$ For 4-cylinder engines $\sum \cos 2\alpha = 2$ and for 6-cylinder $\sum \sin 3\alpha = 3$. For all other equi-angular crank settings the respective summations = 0, consequently the peripheral forces excited by the masses are $P_u = 0$.

II. GENERAL ARRANGEMENT OF THE GEAR.

In the construction of the gear drives for airplane engines the three principal rules of mechanical engineering apply with special force:-

I. All load carrying parts - gearing and housings - must be joined together in the most direct manner to obtain strength and rigidity.

II. Where a variation in distances between centers is unavoidable as a result of wear, or no exact assembly is possible, suitable provision must be made for adjustments.

III. Where a movement in the gears or housing is unavoidable or cushioning or yielding effect is necessary, sliding or elastic joints must be introduced in such a manner that the effect of the movement or yielding can be accurately determined or computed.

For air propeller drives single or double reduction gear drives can be used. The single reduction gears can be either spur gears, or internal gears, and as a rule work out simpler, lighter and cheaper than the double reduction drives. Consequently, they are the most common (Figs. 8 to 11). Practical experience with these gears and inspection of the teeth of their wheels shows that heavy wear takes place in all single reduction gears except the Hispano-Suiza (Fig. 11), in which the teeth generally bear splendidly. The worst bearing is in the teeth of those gears in which the driving pinion is fitted with a bearing on only one side of the wheel.

Single reduction gears with internal gearing have not been captured as yet. However, Birkigt, designer for the Hispano-Suiza Works, has had such a gear patented in England (Fig. 12). A notable feature is the attachment of the internal gear housing to the crank case by an eccentric centering flange which makes it possible to accurately fix the play. Against the great advantages of the internal gear must be balanced the difficulty in arranging satisfactory bearings on both sides of the wheels. However, satisfactory solutions of this problem are not impossible.

Double reduction gears with two different pairs of wheels are principally used where the power must be delivered in the same axial line as the crank shaft. Their construction leads to many and varied solutions, since both pairs of wheels may be fitted with internal or external toothing and in addition any one of the three shafts may be fixed while the other two drive and are driven. In this manner alone 12 solutions are found which are assembled diagrammatically, and for a transmission ratio of 1 : 2, in Fig. 13. The solutions are arranged in the horizontal rows A, B_s, and B_k according to the motion of the intermediate shaft.

Row A Intermediate shaft fixed in housing.
B_s " " revolving with propeller.
B_k " " " " crank shaft.

(Rows B_s and B_k illustrate the planetary gears.) In addition, the vertical rows 1 to 4 are arranged according to the direction of rotation of the shafts. As will be understood all the solutions are not of the same value for actual construction since in different arrangements the provision of bearings on both sides of the gears makes more or less difficulty, the space occupied may be very great and the revolutions of the intermediate shaft may be very high.

For a better comparison the graphical computation has been added in Fig. 13 for each case. According to the well known method the r.p.m. of the crank shaft (n_k) and of the propeller (n_s) have been laid off in magnitude and direction (relative to the fixed housing G) at the point G of the intermediate shaft Z, (Fig. 14). If the spider R is fixed to the housing its r.p.m. is zero and its origin coincides with G. But, if it revolves itself, with r.p.m. n_k or n_s then R is applied at the extremity of n_k or n_s (see Fig. 15). The r.p.m. n_z of the intermediate gears relative to the spider R is assumed as to magnitude and direction as convenient. If now the extremity of n_z is joined to those of n_k and n_s the intersections on the spider indicate the tangent points of the rolling circles.

In addition the double reduction gear can be constructed with spur gears or bevel gears.

The much simpler forms in which both pairs of wheels have one wheel in common form a special case, (see Figs. 16 to 19). The form of Fig. 16 is developed from that of form A_1 of Fig. 13, the form of Fig. 17, from A_4, and of Fig. 18 from B_{s2}. It will be observed with this last that the transmission ratio is limited (to about 1 : 1.5). With the form of Fig. 19 a ratio of only 1 : 2 is possible.

The Rolls-Royce planetary gear is the best known (Figs. 21 to 23). In many details it is directly representative of the type. It is represented by the form B_{s2} of Fig. 13. The gear a revolves with the revolutions + n_k of the crank shaft, gears b and c with the relative revolutions +' n_z in the spider i, which is more plainly shown in Fig. 23, and which in turn revolves with the revolutions n_s of the propeller while gear d is held against revolving in the housing.

The advantage of the double reduction gear over the much simpler single reduction gear lies in the perfectly axial transmission of the power, from which the best condition of loading of the housing - pure torsion - is obtained. If the power is transmitted through 2, 3 or 4 intermediate gears at equal angles springing of the gear shafts from unequal peripheral forces or inaccurate tooth forms does not occur. Certain arrangements also make it possible to use heavy revolving masses, for instance, those of the intermediate shafts or the larger wheels with internal toothing, for the improvement of the uniformity of transmission and to avoid reversals of tooth pressures. The principal advantage, however, consists in the fact that on account of the load being divided between 2 to 4 intermediate gears the tooth pressures per unit of tooth face are very low. Consequently, small pitches and small gears can be used which in turn have smaller construction defects, since the defects resulting from inaccurate dividing wheels increase with increasing radius. The disadvantage of the double reduction gear is that it is relatively heavy and costly and makes great demands on the accuracy or exact adjustment of the intermediate shafts if all the gears are to work equally. Finally, the solid and secure assembly of the gear make necessary a series of connections which do away with the theoretical simplicity of the type.

In accordance with the first law of light machinery construction - that all load carrying parts are to be joined together as rigidly and unvaryingly as possible - it is best to fit only ball bearings in the gear case. Then the possibilities of wear and of changing center distances need not be considered, especially if the gears can be fitted in place with the proper clearance. (Note: Sunbeam fits even the crank shaft gear in plain bearings.)

The crank shafts of most of the engines fitted with gears ran in plain bearings which might wear. This was especially the case with engines having six-throw crankshafts. If a bearing ran hot, probably the centerline of the crankshaft after the engine had been overhauled had another position than originally, unless special means to prevent it were provided. Consequently in the Renault, Peugeot, Napier and similar engines ball bearings were used for the crankshaft and in this manner the difficulties resulting from possible change of center distances were avoided. In the Hispano-Suiza engine only the first crankshaft bearing - which also takes the gear pressure - is a ball bearing. At each overhaul the slightly worn plain bearings must be replaced by new ones truly centered.

If it is not desired to go to these measures it is necessary to fit a joint either in the fixed part, for instance, between crankcase and gear case, (Fig. 24) or in the transmission between crankshaft and gear (Figs. 25 and 27), which will either adjust itself automatically while running or can be adjusted in assembly. If such a joint adjusts itself automatically, as must be the case when it is fitted between crankshaft and gear, it also equalizes the expansions due to heating of the crankcase and gear case and makes the assembly of gear and engine easier. Generally two shafts - or two housings - which are to remain always parallel can be connected by a sliding cross linkage K (Fig. 25), a sort of sliding joint S, (Fig. 24), or a floating shaft W, (Fig. 26).

The method of the sliding cross linkage has been used in the Rolls-Royce gear - however, in a fixed housing - in a notable manner. The link (Fig. 22) and e of Fig. 2 1, lies between the outer engine housing and the intermediate gear wheel, d, which is held in the housing. Consequently this can adjust itself and always remain concentric with the crankshaft. The whole set of planetary gears also always remains concentric with the crankshaft - which may shift in the casing - but not with the casing. The joint between the fixed gear wheel and the housing is accordingly adjustable transversely in any direction and adjusts itself correctly while the forward bearing g and h must be adjusted on each overhaul of the engine by the adjusting screws, f.

In principle this cross link could just as well be placed between crankshaft and gear. In that case, however, the whole gear would have to be carried in rigid bearings in the case. The preceding arrangement saves a bearing and has the advantage that the cross link does not rotate and consequently can be easily kept in oil. Besides it does not have to transmit the varying torque of the crankshaft. In addition the mass of the internal gear which is directly attached to the crankshaft helps the smoothness of running.

For the rest the problem of making the joint in the housing adjustable is best solved either by fitting the gear housing with a flange which is not concentric on the engine housing and which after every overhaul can be adjusted and secured anew, or by making the bearings adjustable (Fig. 27). This new arrangement of the Rolls-Royce gear is known only from patent drawings. In it the upper gear can be adjusted by eccentrically set ball bearing cages, c and d, and the lower gear can be adjusted on the engine shaft by means of adjusting screws. The joint between crankshaft and gear wheel a is a universal one, which is very convenient for assembly.

A gear made by the Friedrichshafen gear factory (Zahnraderfabrik Friedrichshafen) illustrates a method by which the crank shaft can be separated from the rigid gear set, (Fig. 28). At the same time it accomplishes the attachment of large rotating masses to the crankshaft, so as to avoid reversals of pressure, and the separation of the irregularities of the crankshaft from the gear. The utility of this type of construction depends principally on the suitability of the type of coupling used.

When long shafts are used between engine and propeller it is best to fit the joints which have been proven by use in automobiles.

Spring or elastic joints have the advantage that they need no lubrication. They must, however, be absolutely so perfect that their elastic distortion compared to the angular motion allowed is either extremely small or accurately determinate so that their influence on the vibration frequency of the shaft can be determined by experiment or computed. Otherwise they may cause great danger to the security of the gear and engine as a result of the possibility of resonance vibrations.

III. PRIMARY VIBRATIONS IN THE SHAFT TRANSMISSION.

The primary vibration frequency of a freely vibrating crankshaft, resulting from some passing impulse, is determined partly by the masses involved - that is the propeller, the pistons, cranks, counter-weights and gears - and partly by the springing of the shafts and gears. In the usual German 6-cylinder engines, with moderately heavy air propellers, whose moment of gyration lies between 20 and 60 kg/m^2 the freely vibrating shaft has a frequency of about 6000 vibrations per minute; in 4-cylinder engines more, and in the single crank radial or revolving about 20,000. Indeed freely vibrating six-throw shafts make a greater number of vibrations than the figure given, because with the masses distributed on the cranks they can vibrate in two or more nodes instead of one. However, these higher frequencies need never be practically considered.

Various methods can be used for the computation of the vibration frequency; for instance that of Gümbel or that of Kutzbach. (For the former, see "Zeitschrift des Vereines deutscher Ingenieure" 1912, for the latter, the same - 1917.) They can be measured in operation by the use of the Geiger(3) Torsiograph made by Lehman and Michaelis of Hamburg - which requires a degree of practice in its application - or with the engine stopped. For this a series of light blows at regular intervals is applied to the crankshaft. The vibrations thereby excited in the shaft increase markedly when the frequency of the blows coincides with the frequency of primary vibration of the shaft. (When gears are used contact under pressure must be maintained by suitable springs between propeller and crankshaft.) The primary vibration frequency of the crankshaft is of great importance, since in operation a resonant effect from the power impulses of the engine itself absolutely must be avoided. In a 6-cylinder four-cycle engine or a 3-cylinder two-cycle engine there occur regular impulses with frequencies of the threefold, sixfold, ninefold, etc. revolutions, of which the first are the strongest. In an 8-cylinder four-cycle engine or a 4-cylinder two-cycle engine the impulses occur at 2, 4, 6, etc. multiples of the revolutions. Consequently, for a 6-cylinder engine whose crankshaft has a vibration frequency of 6000 per minute the most dangerous speed of revolutions is $6000 \div 3$ or 2000 r.p.m. The next most dangerous is at $6000 \div 6 = 1000$ r.p.m. etc. It is important that the revolutions ordinarily used shall lie as far as possible from the region of resonance.

If the diagrams of the individual cylinders are not equal, that is, if, for instance, in an engine with two carburetors, one-half the engine is regularly delivering more power than the other, then not only are the three and sixfold revolution frequencies, impulse frequencies but also the four and one-half and ninefold revolution frequencies. Thus $6000 \div 4\text{-}1/2 = 1444$ r.p.m. may be dangerous. It is therefore always advisable to keep the primary vibration frequency of the crankshaft of 6-cylinder engines above 6000.

Different builders use elastic couplings between the air propeller masses and the gear masses to improve the uniformity of rotation and protect the gears from torsion. But by that the elasticity increases and the vibration frequency decreases markedly. In a 6-cylinder engine for instance it would be extraordinarily dangerous if the vibration frequency dropped to $3 \times 1400 = 4200$ on account of the use of such a coupling. The shaft revolving at 1400 r.p.m. would break sooner or later. Likewise the gear would have no endurance.

If it is desired or is necessary to use such a coupling it must either be made so unyielding that the vibration frequency will always remain high enough or it must be made so yielding that the primary vibration frequency will not be reached at any ordinary speed and if possible not at idling speed. In any case such couplings should be care-

(3) See Z.V.d.E. 1917.

fully studied out and for safety's sake it should be established that the degree of elasticity in the coupling as actually constructed agrees with that assumed in the computations. Otherwise an endeavor should be made to get along without elastic members.

This question is also of importance in connection with fast running blowers whose masses completely change the vibration frequency of the shafts. For a ratio i the moment of gyration of the blower has an effect on the free vibrations of the i^2 value.

In Figs. 29 to 34 (based on a communication from the Maschinenfabrik Augsburg-Nurnberg, Augsburg) the vibration frequency of the first degree (primary vibration) of an airplane engine and reduction gear is determined graphically. It is assumed that the propeller can be considered as a rigid mass and the gear wheels in actual contact, which is not exactly true. The computation - in which the crankshaft is replaced by a smooth plain shaft of the same value relative to elastic distortion - shows that the primary vibration frequency of the system, consisting of the six driving masses with crankshaft, reduction gear and a certain air propeller, lies at about 6710.

Fig. 35 shows approximately the influence of the different irregular impulses of the motor torque in way of exciting resonance if the crankshaft with a primary vibration frequency of 6710 makes successively from 0 to 2500 r.p.m. It is based on results from the Torsiograph. It can be plainly seen that between 1600 and 2000 r.p.m. the engine is fairly free from critical speeds of rotation and that consequently within this range the gear wheels will be little stressed from vibrations. The critical r.p.m. at $n = 1490$ is only the result of unequal work by the different cylinders, consequently is avoidable and works differently in different cases.

TABLE I. GEARS FROM CAPTURED ENGINES.

No	Engines	N_e HP	$\frac{n_k}{n_s}$	$\frac{z_1}{z_2}$	$\frac{d_1 \text{ cm}}{d_2 \text{ cm}}$	b cm	t mm = m . π	v_u m/sk	P_u kg	$\frac{P_u}{b}$ kg/cm	$\frac{P_u}{b \cdot t}$ kg/cm²	$\frac{P_u}{b} \cdot \frac{n_1+n_2}{z_1}$	δ_r cm	$k = \frac{P_u}{b \cdot \delta_r}$ kg/cm²	Remarks
1	Renault (Air-cooled) 8 Cyl.	70	1650/825	16/32	80/160	3	15.7 = 5 . π	6.9	760	254	162	39300	1.36	187	
2	Renault (Air-cooled) 8 Cyl.	85	1650/825	17/34	93.5/187	3.4	17.28 = 5-1/2 . π	8.1	790	232	134	33800	1.63	142	
3	Renault (Air-cooled) 12 Cyl.	130	1650/825	15/30	93.75/187.5	4.5	19.64 = 6-1/4 . π	8.2	1205	268	134	44250	1.63	164	
4	Renault (de Dion-Bouton) 12 Cyl.	130	1650/825	24/40	90/180	5.2	14.14 = 4-1/2 . π	7.8	1260	242	171	30000	1.55	156	Ratio 1 : 2
5	Engl. Daimler 8 Cyl.	100	2000/1000	18/36	90/180	3.3	15.7 = 5 . π	9.5	800	242	154	40250	1.55	156	
6	Eng. Daimler 12 Cyl.	150	2000/1000	18/36	114.3/228.6	4.0	19.95 = 1/4 . π	12	940	235	118	39200	1.96	120	
7	RAF-Napier 12 Cyl.	250	1800/900	24/48	122/244	3.95	15.97 = 5-1/12 . π	11.5	1630	413	251	46600	2.05	182	
8	Hispano-Suiza	200	2000/1500	21/28	112/149.3	6	16.75 = 5-1/3 . π	11.75	1280	214	127.5	34000	1.65	130	

TABLE I. GEARS FROM CAPTURED ENGINES (Contd.)

No.	Engines	N_e HP	$\frac{n_k}{n_s}$	$\frac{z_1}{z_2}$	$\frac{d_1 \text{ cm}}{d_2 \text{ cm}}$	b cm	$t_{mm} = m \cdot \pi$	v_u m/sk	P_u kg	$\frac{P_u}{b}$ kg/cm	$\frac{P_u}{b \cdot t}$ kg/cm²	$\frac{P_u}{b} \cdot \frac{n_1 + n_2}{z_1}$	δ_r cm	$k = \frac{P_u}{b \cdot \delta_r}$ kg/cm²	Remarks
9	Hispano-Suiza	200	2000/1171	24/41	96/164	6	12.57 $= 4 \cdot \pi$	10.0	1492	249	198	33500	1.55	160	Oblique teeth
10	Hispano-Suiza (Wolsley)	200	2000/1186	35/59	140/236	4.5	12.57 $= 4 \cdot \pi$	14.65	1025	228	181	20750	2.26	100	
11	Rolls-Royce	200	1700/1002	120/45 21/39	240/90 52.5/97.5	2.4 4.5	6.28 $= 2 \cdot \pi$ 7.85 $= 2\text{-}1/2 \cdot \pi$	—	700/1200	97/91	155/116	2800 2500 15400 20000	3.6/0.88	27/104	Large gear an internal gear, number of teeth on each gear divisible by 3.
12	Rolls-Royce	260	1600/1024	120/45 30/45	254/95.2 63.5/95.2	2.5 4.5	$= \frac{1''}{12} \cdot \pi$	—	915/1375	122/103	184/155	2920 2600 8800 17600	3.9/0.98	31/105	
13	Rolls-Royce	300	1650/990	120/45 27/48	254/95.2 57/101.6	2.5 4.5	$= \frac{1''}{12} \cdot \pi$	—	1025/1710	137/127	206/194	3760 3350 13000 22000	3.9/0.94	35/135	

3142-17

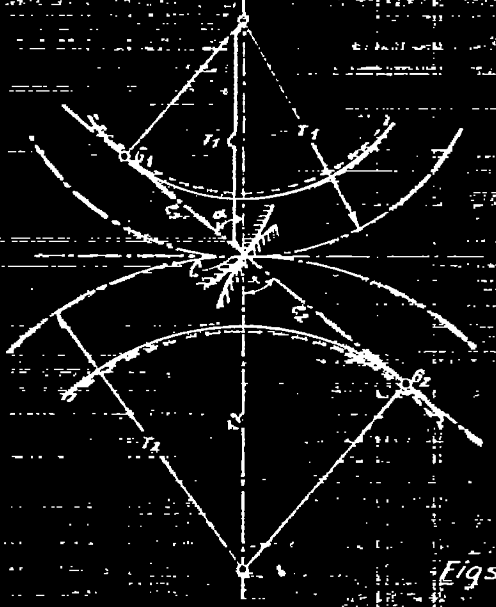


Fig. 1.

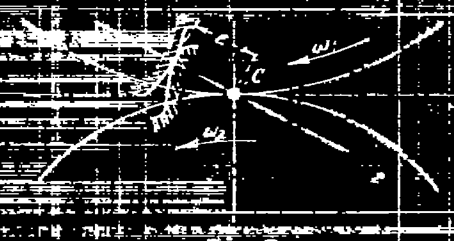


Fig. 2.

Figs. 3 to 7 Diagrams From the Saurer Gear Testing Machine.

Note:

z_1 = No. of Teeth in Pinion.

z_2 = No. of Teeth in Gear.

$\frac{z_1}{z_2}$ = Gear reduction ratio.

m = Module

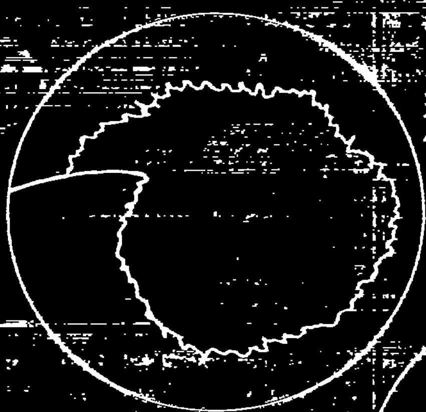


Fig. 3

$\frac{z_1}{z_2} = \frac{13}{47}$, m = 5, ground, hardened

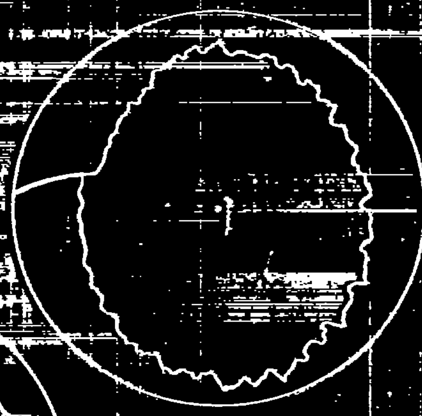


Fig. 4.

$\frac{z_1}{z_2} = \frac{20}{40}$, m = 5, ground, hardened.

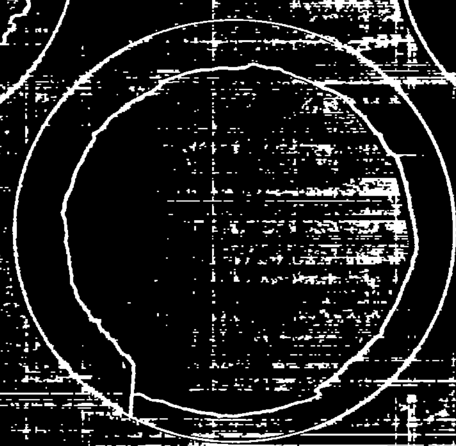


Fig. 7.

$\frac{z_1}{z_2} = \frac{12}{12}$, m = 8,5. Grinding objectionable.

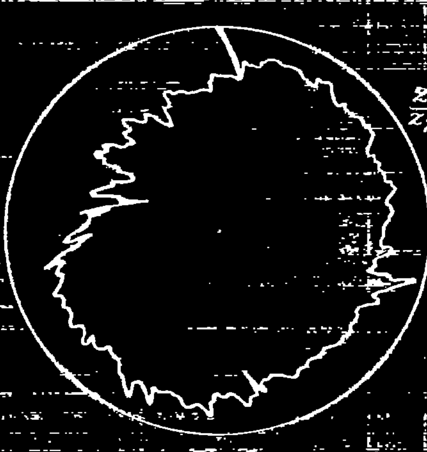


Fig. 5

Cut and hardened, before grinding.

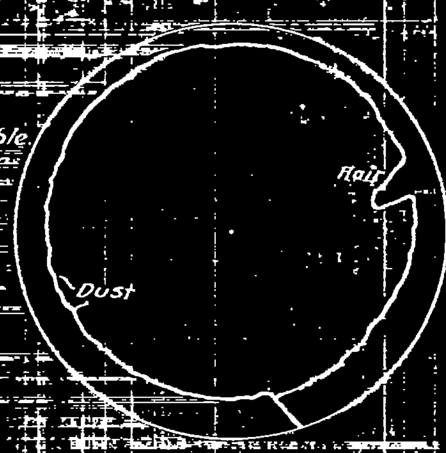


Fig. 6.

Hardened and ground (not objectionable)

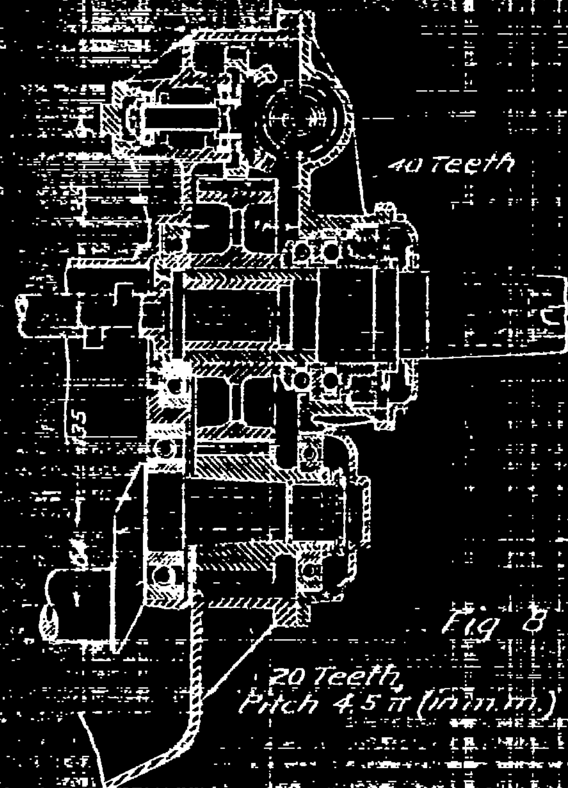


Fig. 8

Gear drive of a 130 H.P. Renault Engine
built by de Dion & Bouton.

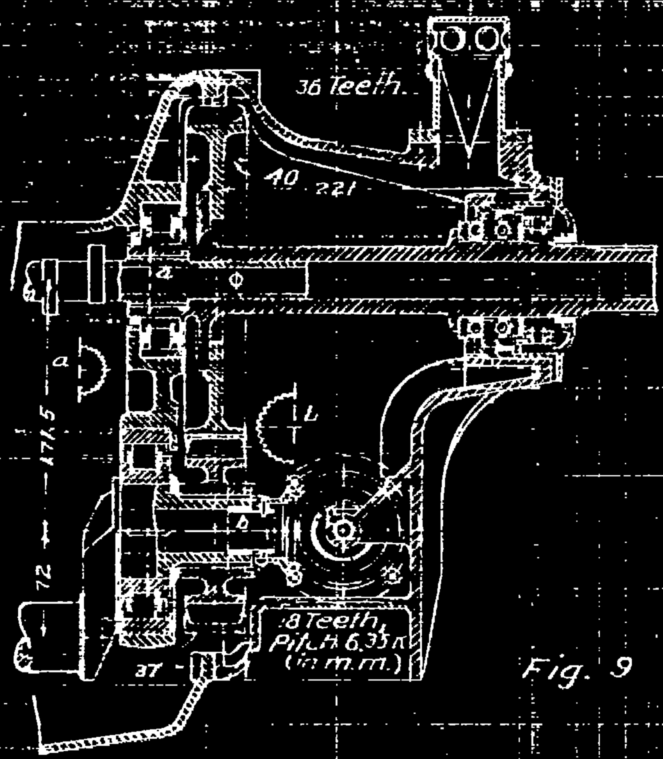


Fig. 9

Gear from a 150 H.P. R.A.F. Engine
built by the British Daimler Co.

Fig. 8 & 9

3142-11

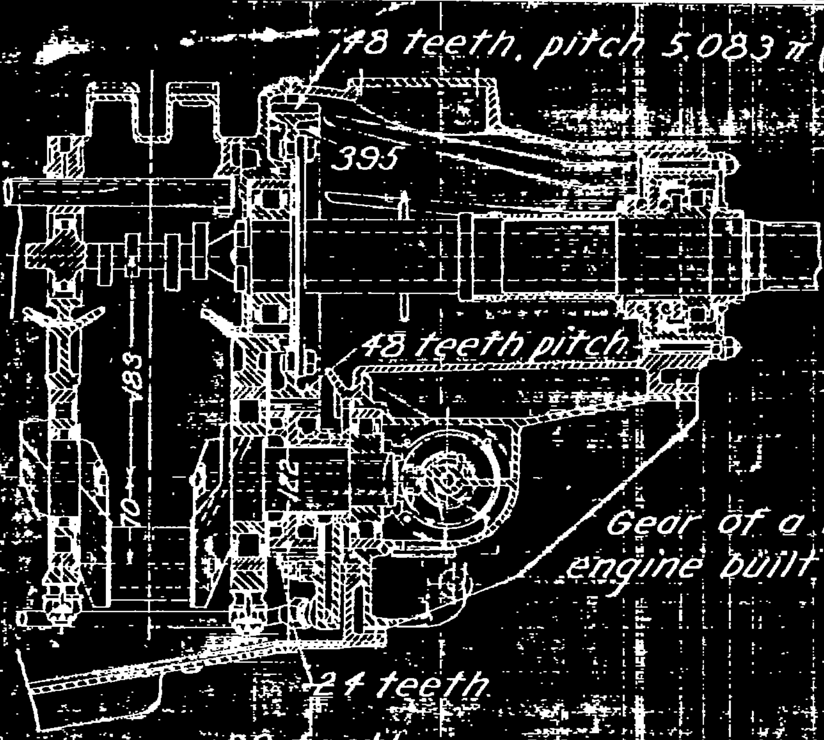


Fig. 10.
Gear of a 200 H.P. R.A.F. engine built by Napier.

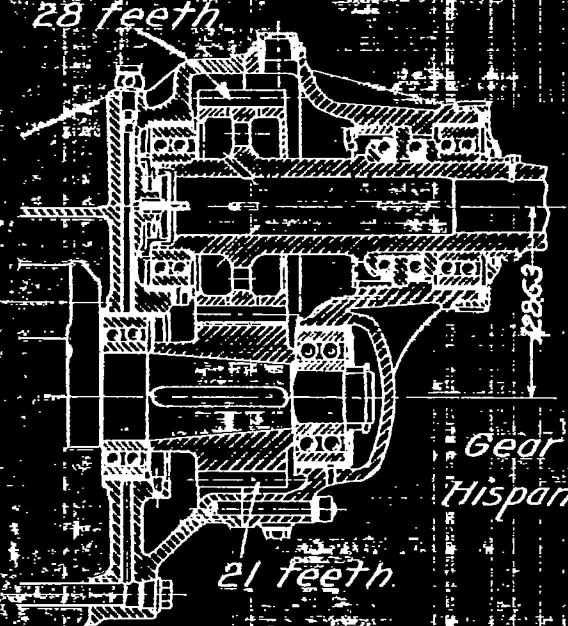


Fig. 11.
Gear of a 200 H.P. Hispano Suiza engine.

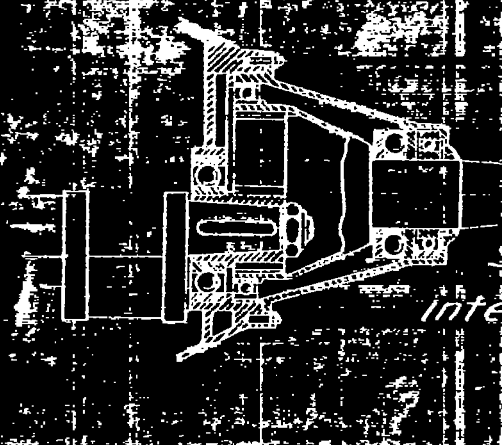


Fig. 12.
Single reduction internal gear by Birkigt

Fig. 10, 11 & 12

Crank shaft (n_k) and propeller (n_s) turning in the same direction.		Crank shaft and propeller turning in opposite directions.	
Intermediate shaft (n_z) turning in opposite direction.	Intermediate shaft turning in same direction as crank shaft.	Intermediate shaft turning in same direction as crank shaft.	Intermediate shaft turning in opposite direction.

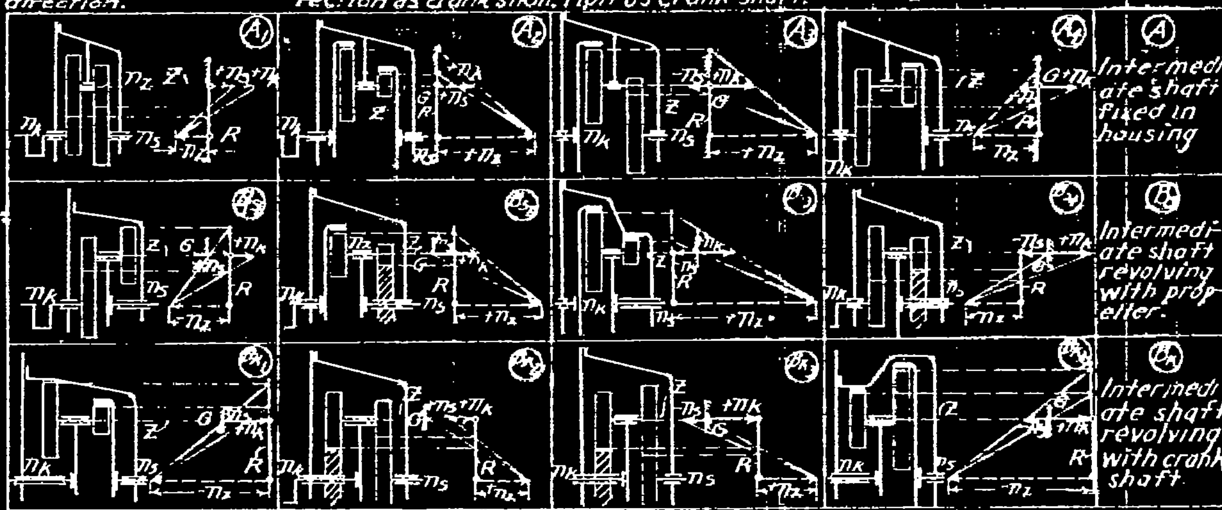


Fig. 13.

Various arrangements of double reduction gears.

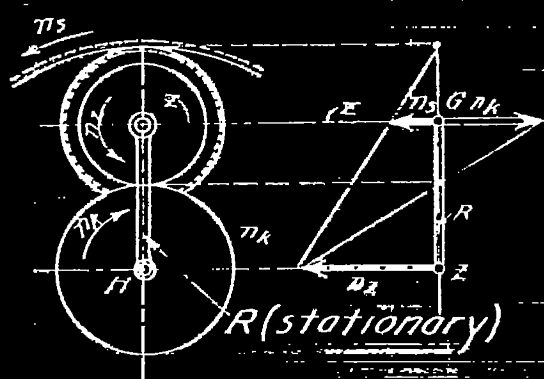


Fig. 14.

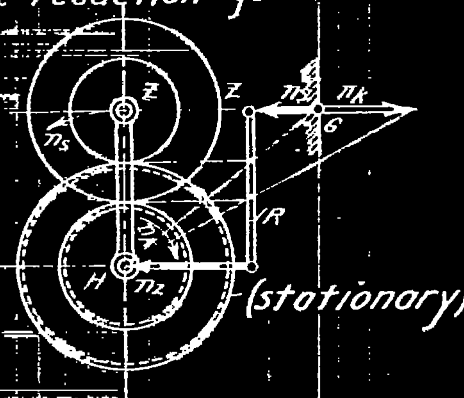


Fig. 15.

Fig. 16.

Fig. 17.

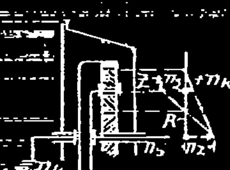


Fig. 18.

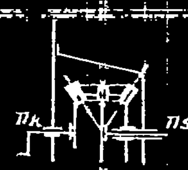


Fig. 19.

Double reduction gear in which the pairs of gears have a common gear.

Figs. 13, 14, 15, 16, 17, 18 & 19.

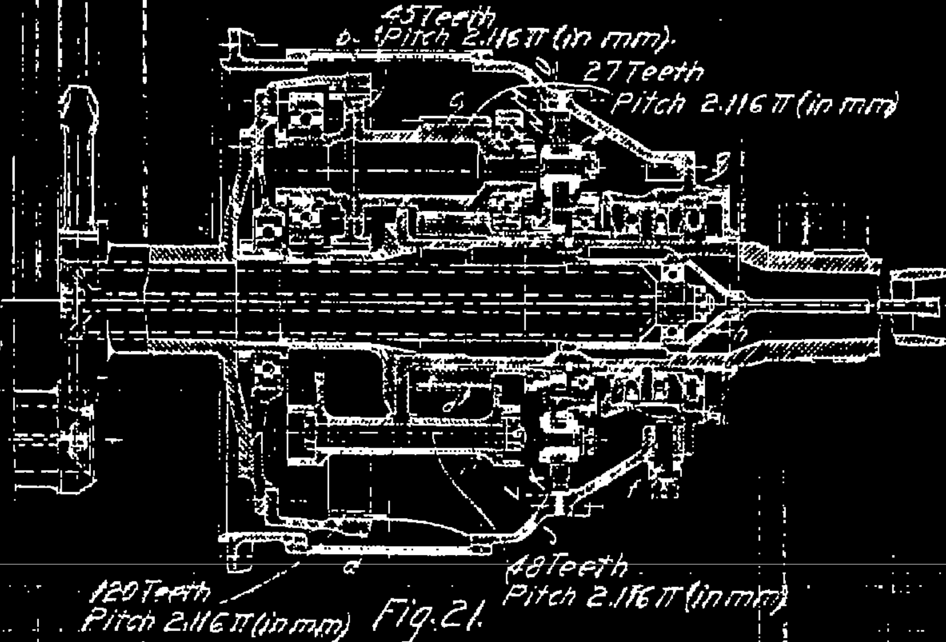


Fig. 21.

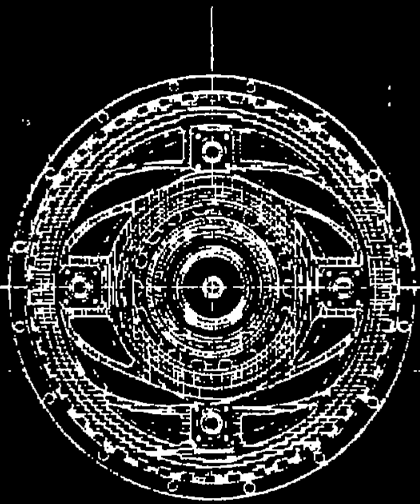


Fig. 22.

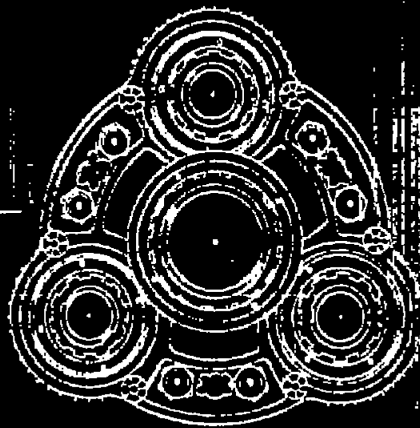


Fig. 23.

Section on J.K.L.M.N.O.

ROLLS ROYCE PLANETARY GEAR.

Figs. 21, 22 and 23.

3142-11

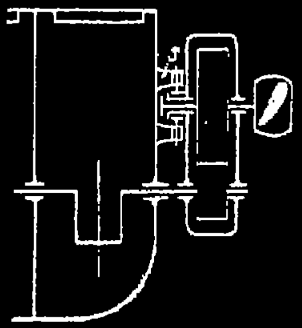


Fig. 24.

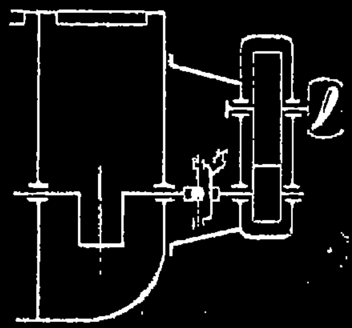


Fig. 25.

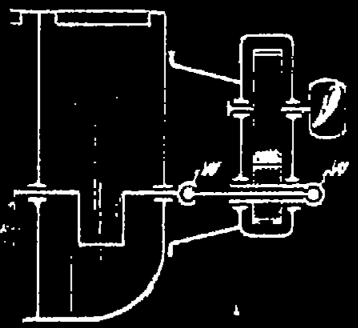


Fig. 26.

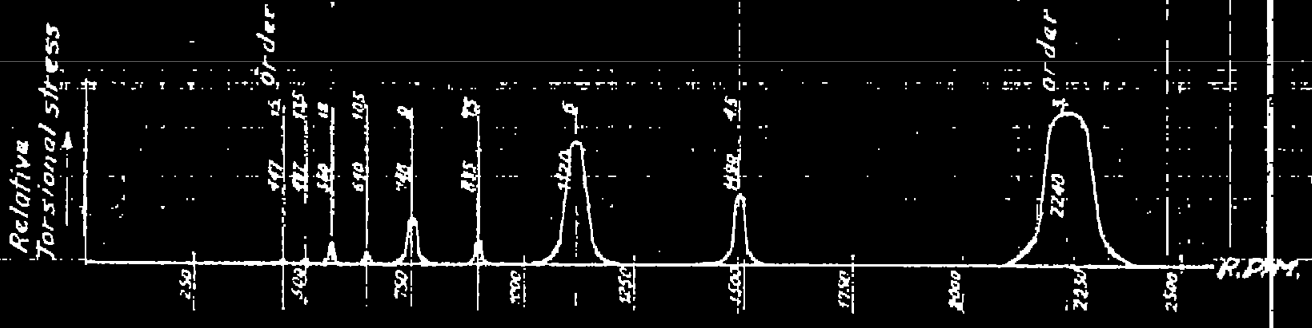


Fig. 35. Dependence of the torsional stress on the speed of rotation.

Figs. 24, 25, 26 & 35.

9/142-11

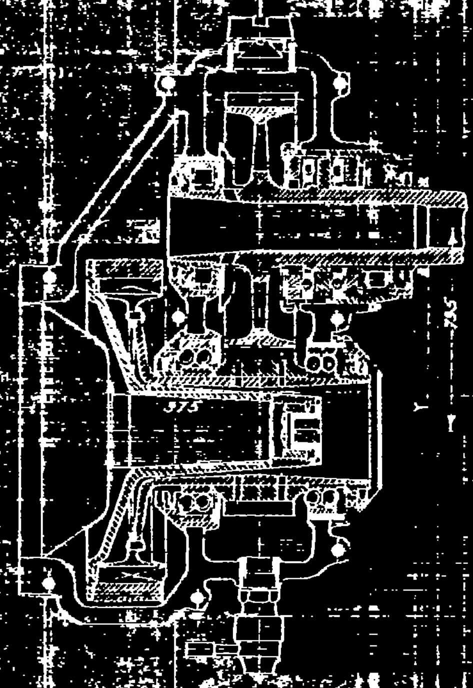


Fig. 28 (The Friedrichshafen Gear Factory Gear.)

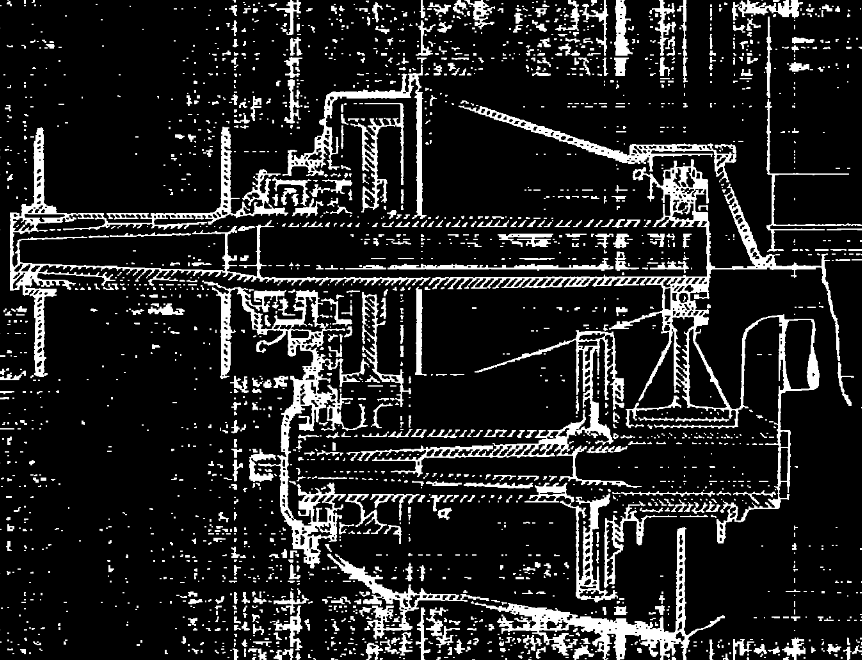
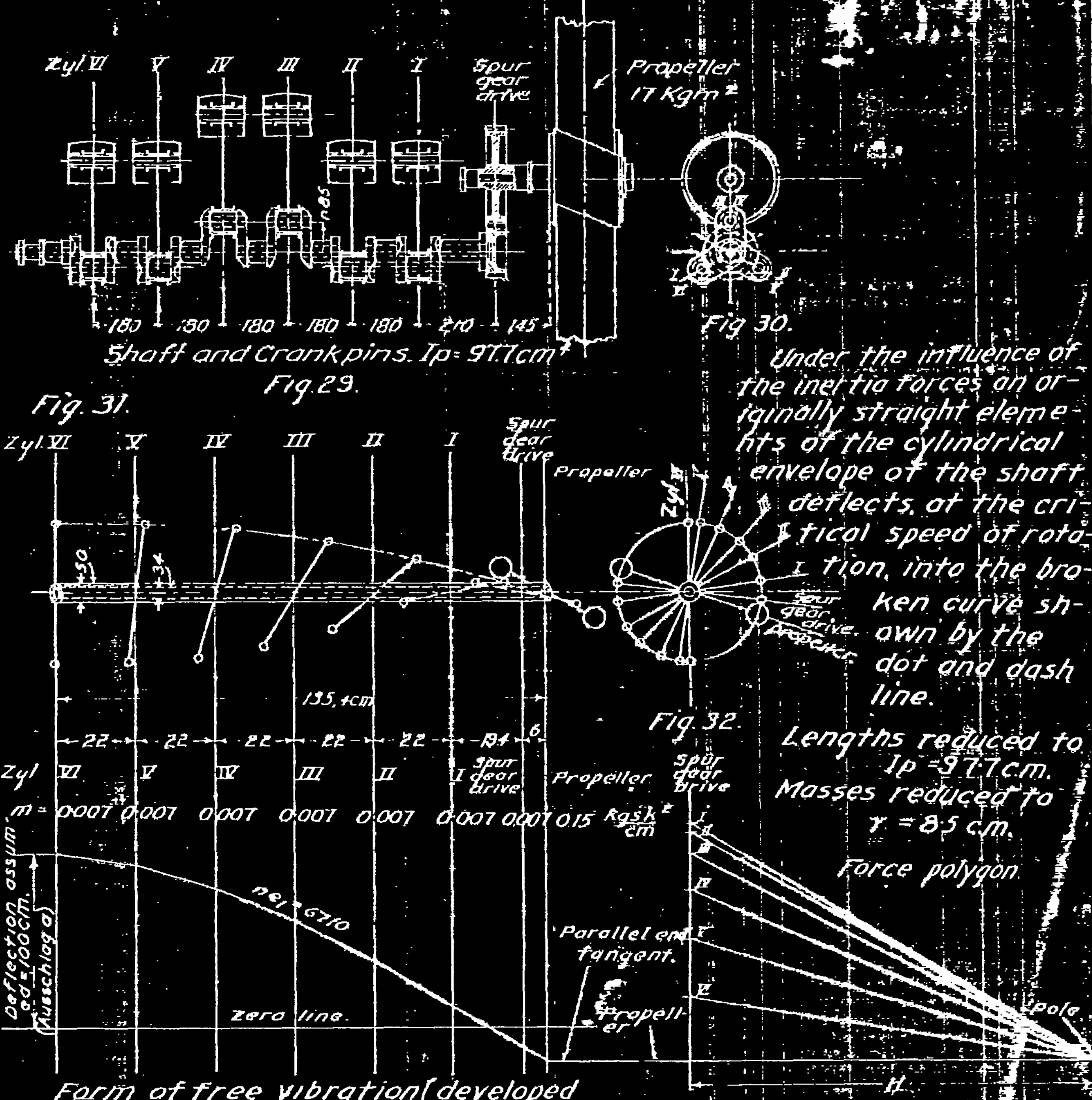


Fig. 27. Rolls-Royce Spur Gear.

Figs. 29 to 35 Computation of the torsional primary vibration frequency of an aircraft engine.



Shaft and Crankpins. $I_p = 97.7 \text{cm}^4$

Fig. 29.

Under the influence of the inertia forces an originally straight elements of the cylindrical envelope of the shaft deflects, at the critical speed of rotation, into the broken curve shown by the dot and dash line.

Lengths reduced to $I_p = 97.7$ cm. Masses reduced to $r = 8.5$ cm.

Form of free vibration (developed into a plane) for the primary vibration frequency of 6710.

Fig. 33.

$$H = \frac{I_p \cdot G}{r_1^2} = \frac{97.7 \cdot 828 \cdot 10^4}{8.5^2} = 112 \cdot 10^4 \text{ kg}$$

Fig. 34.

Figs. 29, 30, 31, 32, 33, 34